Timed automata

Limitation with LTSs

- They allow to express sequences of actions, choices, loops, and concurrency
- But they cannot model time and time-dependent constraints
- Time is essential in many real-world scenarios
  - Railway systems, e.g.: a crossing barrier takes x seconds to get lowered, and must be lowered y seconds before the train arrives
  - Embedded controllers, e.g.: a safety system in a power plant must react within x seconds
- Idea: extend LTSs by adding time

Timed LTS (TLTS)

A TLTS is a 6-ple $\langle S, A, \Delta, T, \Theta, s_0 \rangle$

- S: States, A: Actions, $T \subseteq S \times A \times S$: Labelled transition relation, s_0: Initial state (like LTS)

- Δ : Time domain
  - Usually, $\Delta = \Re^{\geq 0}$ (real numbers ≥ 0)

- $\Theta \subseteq S \times \Delta \times S$: Timed transition relation
  - $s \xrightarrow{t} s'$: From state s, the system can reach state s' by waiting for a time t

TLTS: Constraints on Θ

We need to introduce these constraints so that the TLTS "makes sense" (i.e., it respects our intuitions about time)

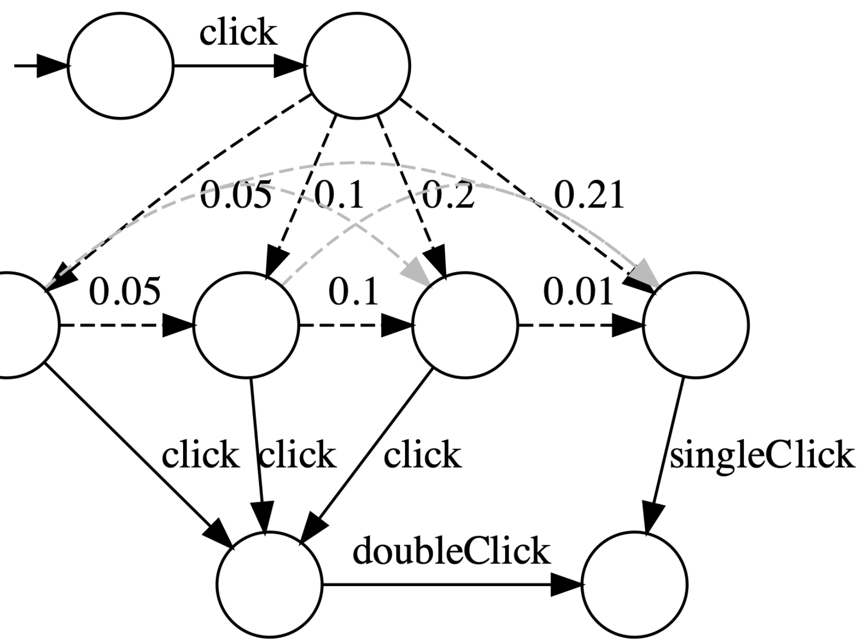
- Time determinism
  - If $s \xrightarrow{t} s'$ and $s \xrightarrow{t} s''$ then $s' = s''$
  - Waiting cannot lead to different states
- Time additivity
  - If $s \xrightarrow{t_1} s'$ and $s' \xrightarrow{t_2} s''$ then $s \xrightarrow{(t_1+t_2)} s''$
  - Waiting t_1 and then t_2 is the same as waiting (t_1+t_2)

Representation of TLTSs (1/2)

- We were able to represent LTSs as graphs with labelled edges. We cannot give a similar, graphical representation of TLTS
- Let's try anyway…
- Example: double click in a GUI
  - At time $t = 0$, user clicks the mouse button.
  - If user clicks the button again while $t \leq 0.2s$, the computer registers a double click
  - Otherwise, the computer registers a single click

Representation of TLTSs (2/2)

- The user can do the 2nd click at any moment in that 0.2 seconds timespan
- Θ and S will have an infinite (non-countable) number of elements!

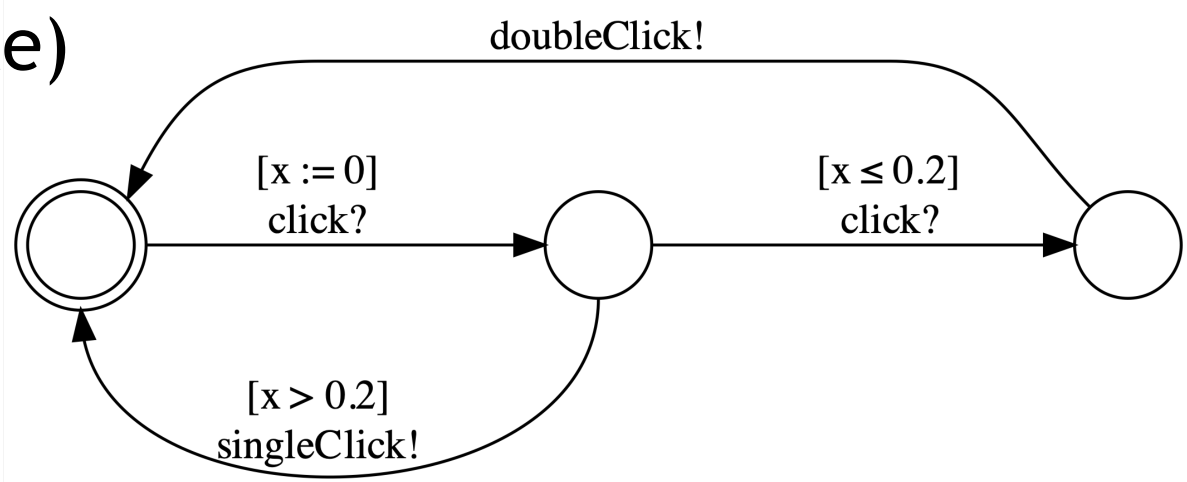


Timed automata

- A "compact" formalism to describe TLTSs
- Communicating automata + clocks
  - Clocks = variables whose values increase continuously
  - The values of all clocks increase at the same speed
  - Can be tested: is the value of c ($\leq, \geq, =, \neq$) some value?
  - Can be reset to 0
- Software support: Uppaal www.uppaal.org

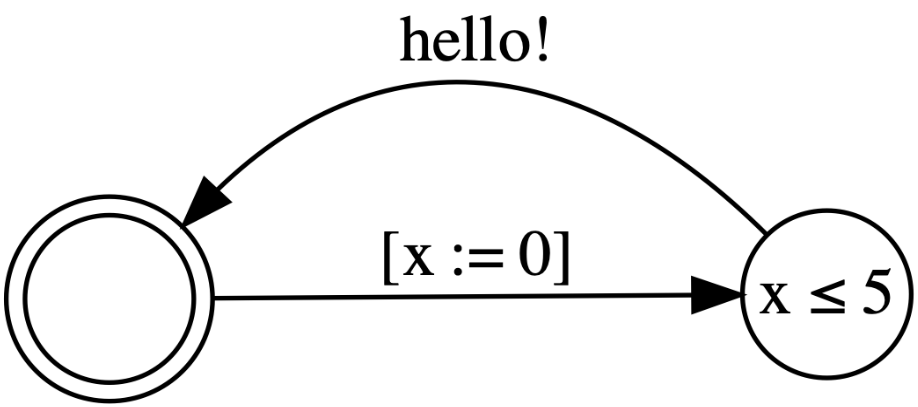
TA example: double click

- ? and ! denote input and output actions
- x is a clock
  - 1st click resets x ($x := 0$)
  - If a 2nd click happens while $x \leq 0.2$, a double click is registered
  - Otherwise, a single click is registered
- (◎ : initial state)



Clock conditions (1/2)

- Guards (Attached to transitions)
  - The transition is enabled iff. the guard is satisfied
- Invariants (Attached to states)
  - The invariant is true as long as the system stays in that state
  - Example: this TA ouputs "hello" before $x > 5$



Clock conditions (2/2)

- A condition can be:
  - A comparison of the value of a clock x with a constant c
  - A comparison of $(x - x')$ with c
  - A negation (NOT) of a condition, or a conjunction (AND) or disjunction (OR) of conditions

$$\Psi ::= x \ \mathbf{op} \ c \mid x - x' \ \mathbf{op} \ c \mid \neg\Psi \mid \Psi \wedge \Psi \mid \Psi \vee \Psi$$

$$op ::= < \mid > \mid \leq \mid \geq \mid = \mid \neq$$

TA: Definition

A TA is a 6-ple $\langle S, A, X, T, Inv, s_0 \rangle$

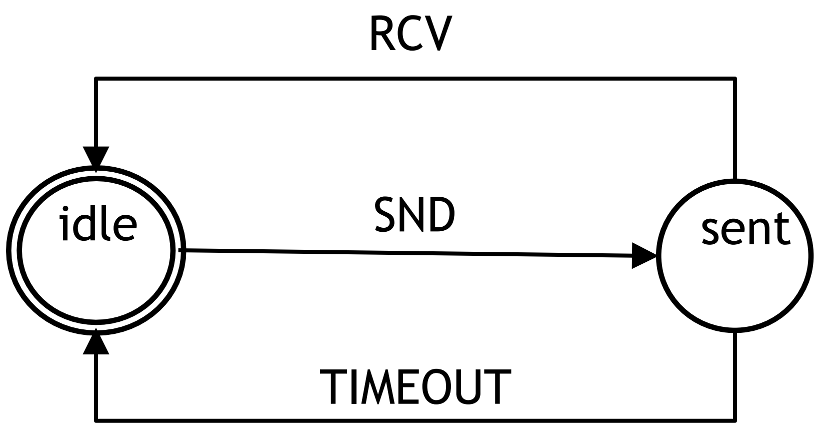
- S: States, A: Actions, s_0: Initial state (like LTS)

- X: Set of clocks
- T : Transition relation: set of 6-ples (s, a, g, r, s')
  - s, s': source and target states
  - $a \in A$: action
  - $g \in \Psi$: a guard over clocks
  - $r \subseteq X$: a subset of clocks that will be reset
- Inv : S $\rightarrow \Psi$ maps each state to an invariant
- All sets are finite

Exercise: Communication medium with timeout

Complete the following CA to make a TA such that:

- Action RCV can occur between 1 and 4 TU after action SND
- If action RCV has not occurred after 4 TU, then action TIMEOUT occurs within 1 TU



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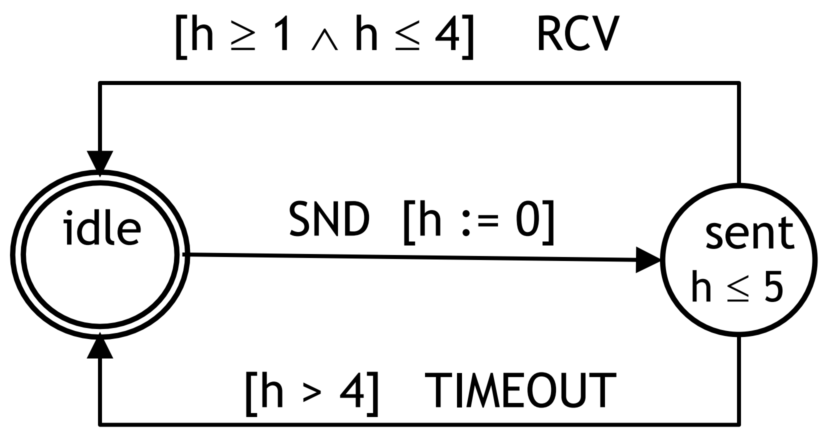
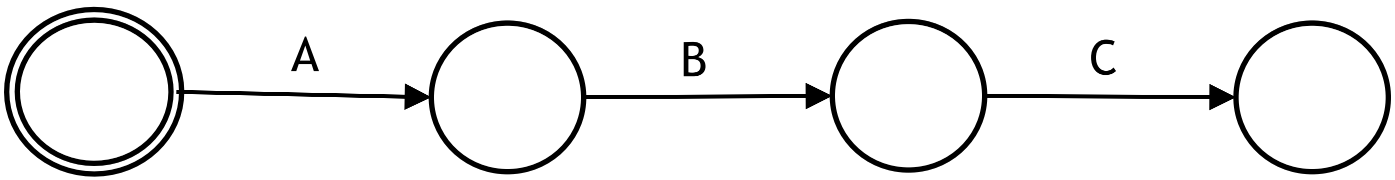


Exercise

Complete the following CA to make a TA such that:

- Action B occurs between 2 and 4 TU after action A
- Action C occurs at least 4 TU after action A and at least 1 TU after action B

Hint: use two clocks



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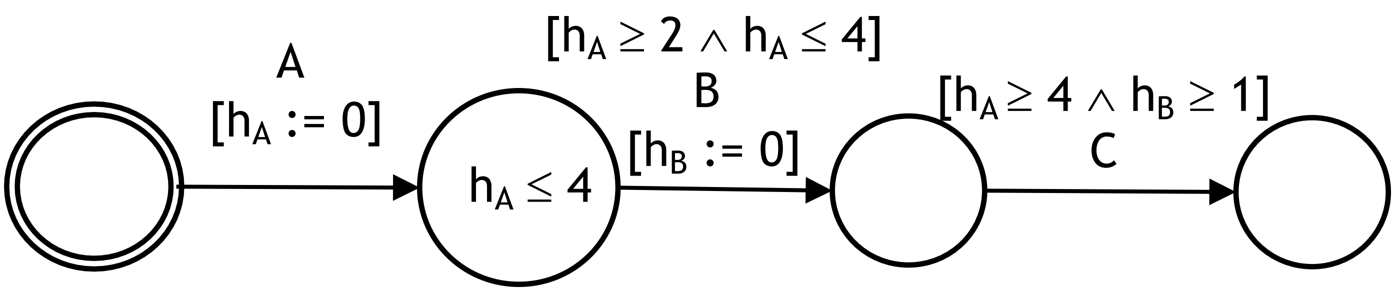


Semantics of TA (1/2)

- General idea: associate a TLTS to every TA

$$\text{TA} = \langle S, A, X, T, Inv, s_0 \rangle$$

$$\textbf{TLTS} = \langle S \times V, A, \Re^{\geq 0}, T', \Theta, (s_0, v_0) \rangle$$

- States of TLTS = (States of TA) × (clock valuation)
  - A valuation $v : X \rightarrow \Re^{\geq 0}$ is a function that assigns a value to every clock. V is the set of all valuations
  - v_0 is the valuation such that all clocks are set to 0.
  - $v+t$ $(t \in \Re^{\geq 0})$ is the valuation v' where all values in v are increased by t time units: $\forall x \in X.\ v'(x) = v(x) + t$
- Initial state of TLTS: (s_0, v_0)

Semantics of TA (2/2)

- T' (discrete transitions): $(s, v) —a\rightarrow (s', v')$ iff.
  - TA contains a transition (s, a, g, r, s')
  - Valuation v satisfies the guard g
  - All clocks in r are reset to 0 in v', while all other clocks have the same value in v and v'
  - v' satisfies the invariant $Inv(s')$

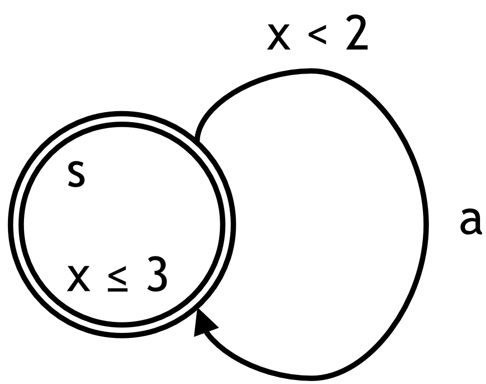
- Θ (timed transitions): $(s, v) —\text{t}\rightarrow (s, v+t)$ iff.

all valuations between v and $v+t$ satisfy $Inv(s)$

  - $\forall\, dt \in [0, t]\ .\ v+dt \vDash Inv(s)$

Timelock

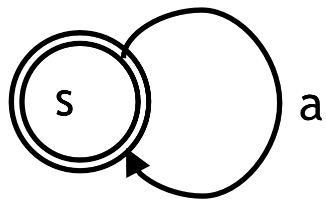
- May arise from using invariants incorrectly
- Example:



- What happens when $x = 3$?
  - Clock x is never reset: time stops
- Unacceptable! Either reset x, or add other edges/states describing what happens when $x = 3$
- Can be detected automatically via verification

Critical paths and Zeno effect

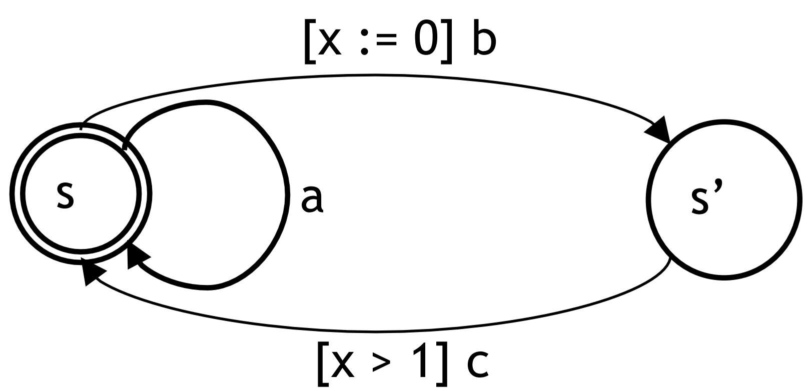
- Example:



- Critical path: infinite actions in zero time
  - $(s, \emptyset) \text{---} a \rightarrow (s, \emptyset) \text{---} a \rightarrow (s, \emptyset) \text{---} a \rightarrow \ldots$
- Zeno effect: infinite actions in finite time
  - $(s, \emptyset) \text{---} a \rightarrow (s, \emptyset) \text{---} 1/2 \rightarrow (s, \emptyset) \text{---} a \rightarrow (s, \emptyset) \text{---} 1/4 \rightarrow \ldots$
  - Will perform an infinity of a actions in 1 time unit
- These kinds of paths are generally allowed, but it's good to prove that time passes (there are paths that are not critical/Zeno)

Time progress

- For some time interval t and some n, every state of the TLTS admits at least one path of length $\leq n$ such that at least t time units pass
- The system may still contain critical/Zeno paths
- Example:
  - "aaa…" path is critical
  - "bc" path takes at least 1 time unit



Parallel composition of TA (1/2)

- Same idea as with CA: we want to decompose complex (timed) systems into small components
- Again, rendez-vous on pairs of actions according to a synchronization set *L*
  - Symmetrical (same actions)
  - Asymmetrical (input/output pairs) (e.g., Uppaal)
- But we also have to take into account:
  - Guards
  - Resets
  - Invariants

Parallel composition of TA (2/2)

- $TA_1 = \langle S_1, A_1, X_1, T_1, Inv_1, s_{01} \rangle$,
- $TA_2 = \langle S_2, A_2, X_2, T_2, Inv_2, s_{02} \rangle$ with $X_1 \cap X_2 = \emptyset$
- $L \subseteq A_1 \cap A_2$ (synchronization actions)

Then,

$TA_1 \otimes_L TA_2 = \langle S_1 \times S_2, A_1 \cup A_2, X_1 \cup X_2, T, Inv, (s_{01}, s_{02}) \rangle$

- $Inv(s_1, s_2) = Inv_1(s_1) \wedge Inv_2(s_2)$
- T:

$$\frac{s_1 \xrightarrow{g,a,r} s_1' \quad a \notin L}{(s_1, s_2) \xrightarrow{g,a,r} (s_1', s_2)} \qquad \frac{s_2 \xrightarrow{g,a,r} s_2' \quad a \notin L}{(s_1, s_2) \xrightarrow{g,a,r} (s_1, s_2')}$$

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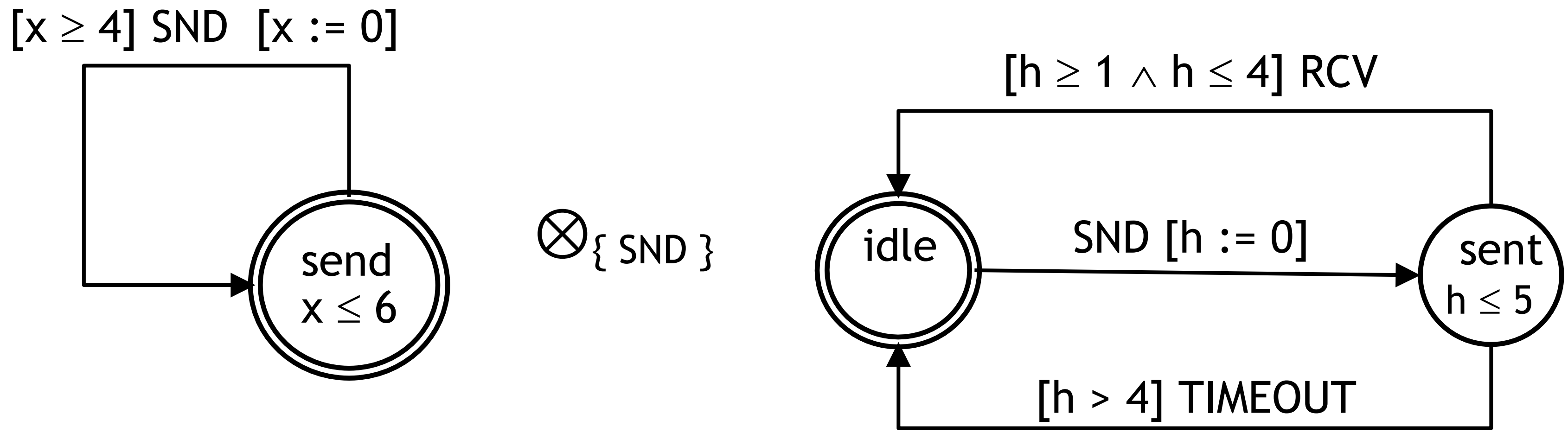
Exercise



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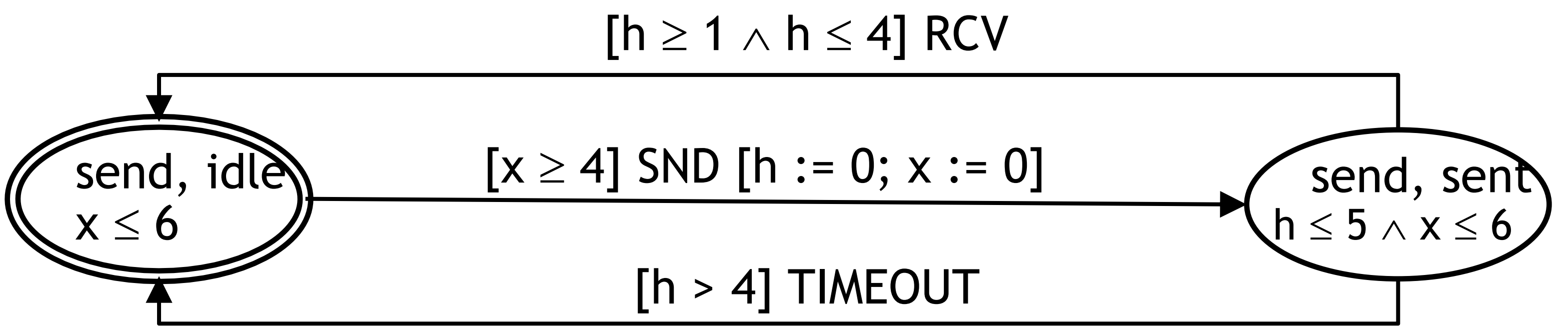

Conclusions

- TA allow to describe systems where time matters
- This introduces additional complexities
  - Underlying model (TLTS) has uncountably ∞ states and transitions
  - Timelocks, critical paths, Zeno effect…
- We can compose TAs via a product $\otimes$
- Automated tools can verify several aspects related to TA correctness