

# A Model Checking Language for Concurrent Value-Passing Systems

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# Context

- **Concurrent systems**

- Process algebraic languages (LOTOS, muCRL)
- Value-passing communication
- Interleaving semantics (LTSs)
- Branching-time world (adequate with bisimulations)

- **Explicit-state, on-the-fly verification**

- Enumeration of individual states and transitions
- Incremental construction of the LTS

- **CADP toolbox**

<http://www.inrialpes.fr/vasy/cadp>



# Outline

- Standard  $\mu$ -calculus
- The MCL language
- EVALUATOR 4.0: an implementation
- Demo (analysis of the SCSI-2 protocol)
- Conclusion and future work



# Standard modal $\mu$ -calculus

- **Assembly** language for temporal logics
- **Models:** Labelled Transition Systems (LTSs)
- Very **simple grammar:**

$P ::=$	$\mu X . P$		$A ::=$	$\text{not } A$
	$\nu X . P$			$A_1 \text{ or } A_2$
	$\langle A \rangle P$	state formula		$A_1 \text{ and } A_2$
	$[ A ] P$			true
	$X$			false
	$\text{not } P$	action formula		label
	$P_1 \text{ or } P_2$			
	$P_1 \text{ and } P_2$			
	true			
	false			

Annotations:

- propositional variable (points to  $X$ )
- state formula (points to  $\langle A \rangle P$ )
- action formula (points to  $\text{not } P$ )



# Standard $\mu$ -calculus: modalities

• **Possibility:**  $\langle A \rangle P$

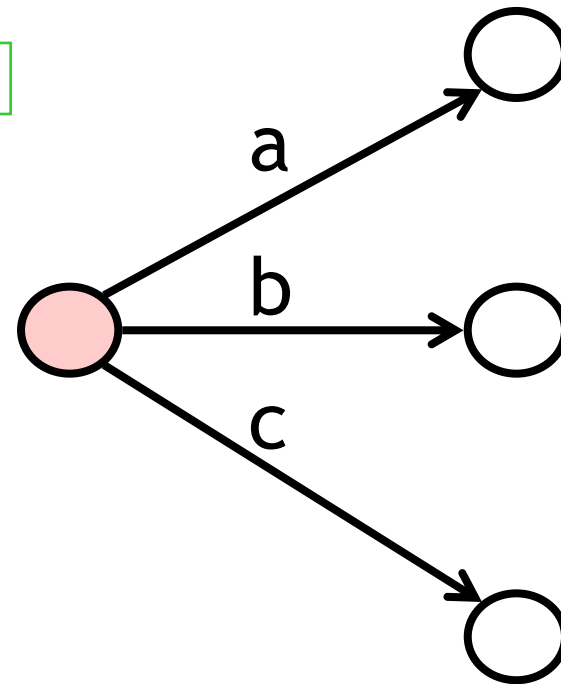
state formula

action formula

$\langle a \rangle \text{true}$

• **Necessity:**  $[A] P$

$[d] \text{false}$

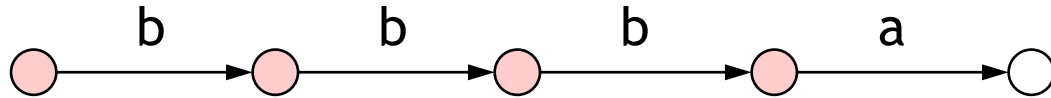


# Standard $\mu$ -calculus: fixed points

• **Intuition:** « recursive functions » on the LTS

• **Minimal fixed point:**  $\mu X . P(X)$

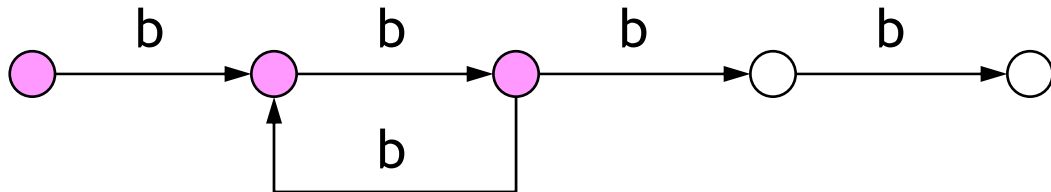
→ characterizes finite tree-like patterns



$\mu X . \langle a \rangle \text{ true or } \langle \text{not } a \rangle X$

• **Maximal fixed point:**  $\nu X . P(X)$

→ characterizes infinite tree-like patterns (cyclic subgraphs)



$\nu X . \langle b \rangle X$



# Syntactic restrictions

- Syntactic monotonicity [Kozen-83]

- Necessary to ensure the existence of fixed points
- In every formula  $\mu X . P(X)$ , every free occurrence of  $X$  in  $P$  falls in the scope of an even number of negations

$\mu X . \langle a \rangle X$  or  $\text{not } \langle b \rangle X$



- Alternation depth 1 [Emerson-Lei-86]

- Necessary for efficient (linear-time) verification
- In every formula  $\mu X . P(X)$ , every maximal subformula  $\nu Y . P'(Y)$  of  $P$  is closed

$\mu X . \langle a \rangle \nu Y . ([b] Y \text{ and } [c] X)$



# Extending $\mu$ -calculus

- Temporal logics (CTL, PDL, ...) and  $\mu$ -calculi
  - No data manipulation (basic LOTOS, ACP)
  - Too low-level operators (complex formulas)

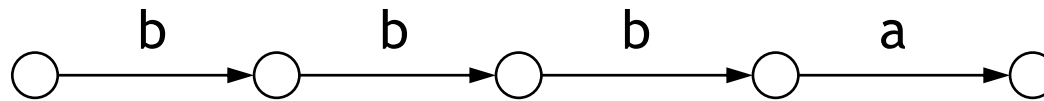
→ *extended temporal logics are needed*
- EVALUATOR 3.5
  - On-the-fly model checker included in CADP
  - Standard  $\mu$ -calculus + regular operators ( $\cdot$ ,  $|$ ,  $*$ ,  $+$ )
  - Diagnostic generation, libraries of derived operators
  - Extensively used and tested (30 case-studies)
  - *No data handling*





# Why do we want to handle data?

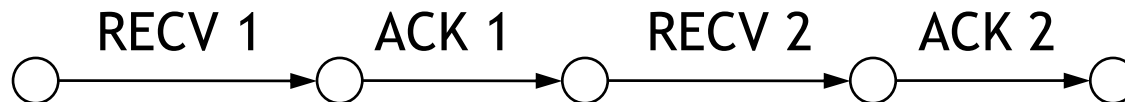
## • Action counting



→ *cumbersome*:  $\langle b \rangle \langle b \rangle \langle b \rangle \langle a \rangle \text{true}$

## • Synchronization with data exchange

- Common in process algebras
- Parameterized LTS (PLTS)
  - label = action + data



→ *temporal properties on PLTSs require data value extraction*

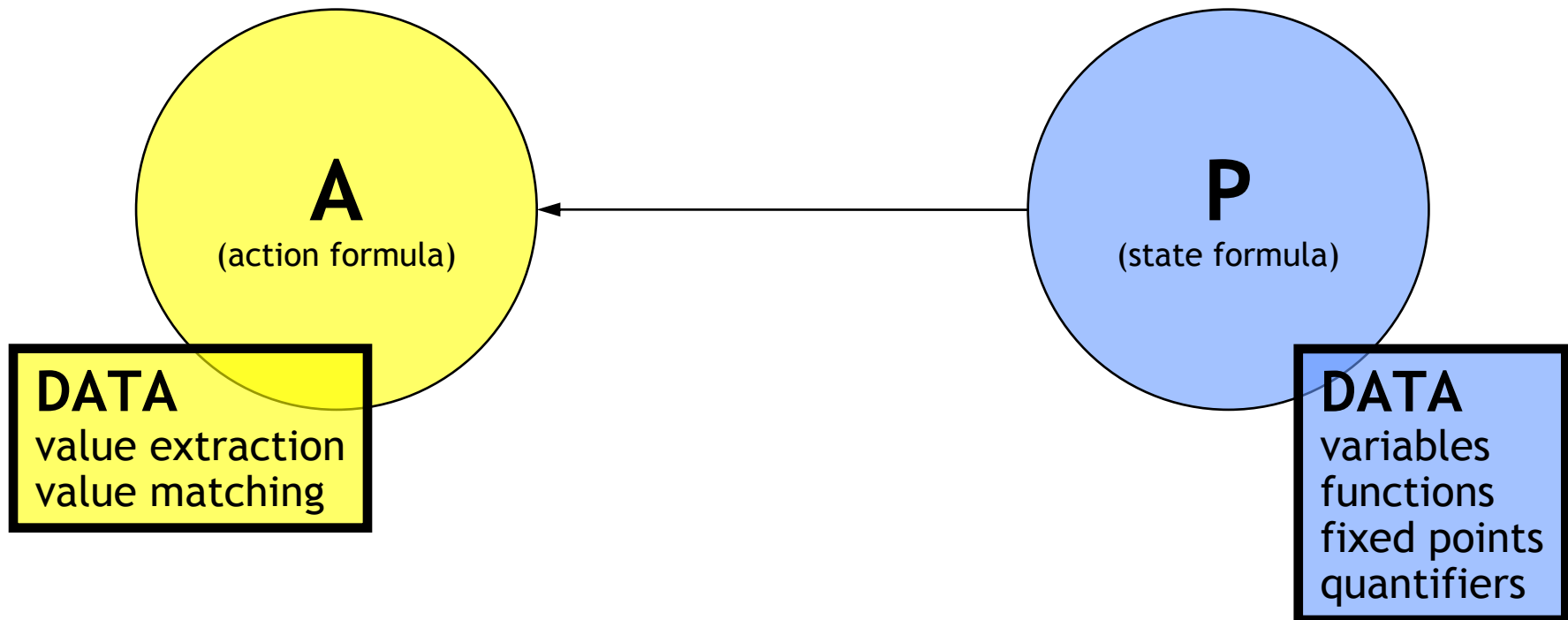


# New **M**odel **C**hecking **L**anguage

- Based on EVALUATOR 3.5 input language
  - standard  $\mu$ -calculus
  - regular operators
- Data-handling mechanisms
  - data extraction from PLTS labels
  - regular operators with counters
  - variable declaration
  - expressions
- Constructs inspired from programming languages



# Model Checking Language (1/4)



# Grammar extension

$P ::= \mu X (y_1:T_1:=E_1, \dots, y_n:T_n:=E_n) . P$   
 $\nu X (y_1:T_1:=E_1, \dots, y_n:T_n:=E_n) . P$   
 $[ A ] P$   
 $\langle A \rangle P$   
 $X (E_1, \dots, E_n)$   
 $y$   
 $f (E_1, \dots, E_n)$   
 $\text{let } y:T:=E \text{ in } P$   
 $\text{not } P$   
 $P_1 \text{ or } P_2$   
 $P_1 \text{ and } P_2$   
 $\text{true}$   
 $\text{false}$   
 $\text{forall } y:T \text{ among } \{E_1 \dots E_2\} \text{ in } P$   
 $\text{exists } y:T \text{ among } \{E_1 \dots E_2\} \text{ in } P$

parameterised  
fixed point

parameterised  
propositional variable

data variable

function call

variable declaration

quantifiers

$A ::= \text{not } A$

$A_1 \text{ or } A_2$

$A_1 \text{ and } A_2$

$\text{true}$

$\text{false}$

$\text{label}$

$\{ G !v_1 \dots !v_n \}$

$\{ G ?x_1:T_1 \dots ?x_n:T_n \}$

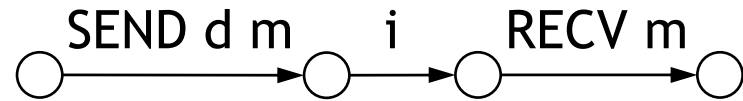
value  
matching

value  
extraction



# Parameterised action formulas

- Parameterised LTS:



- (basic) syntax:

$\{G !E_1 \dots !E_n\}$

*value matching*

gate

expressions

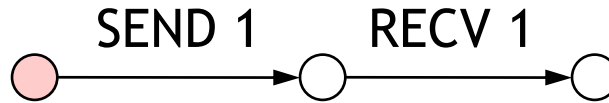
data variables

$\{G ?x_1:T_1 \dots ?x_n:T_n\}$

*value extraction*

# Parameterised modalities

- Possibility:



$\langle \{\text{SEND } ?\text{msg:Nat}\} \rangle \langle \{\text{RECV } !\text{msg}\} \rangle \text{true}$



- Necessity:

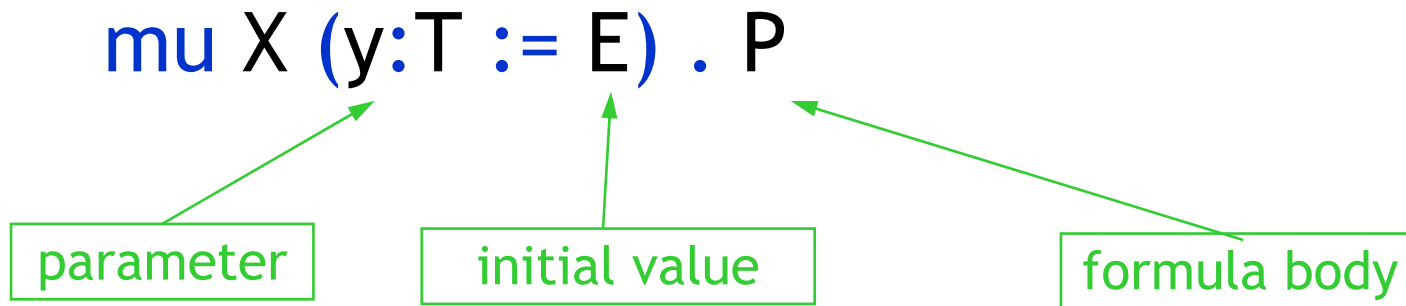


$[ \{\text{RECV } ?\text{msg:Nat}\} ] (\text{msg} < 6)$



# Parameterised fixed points

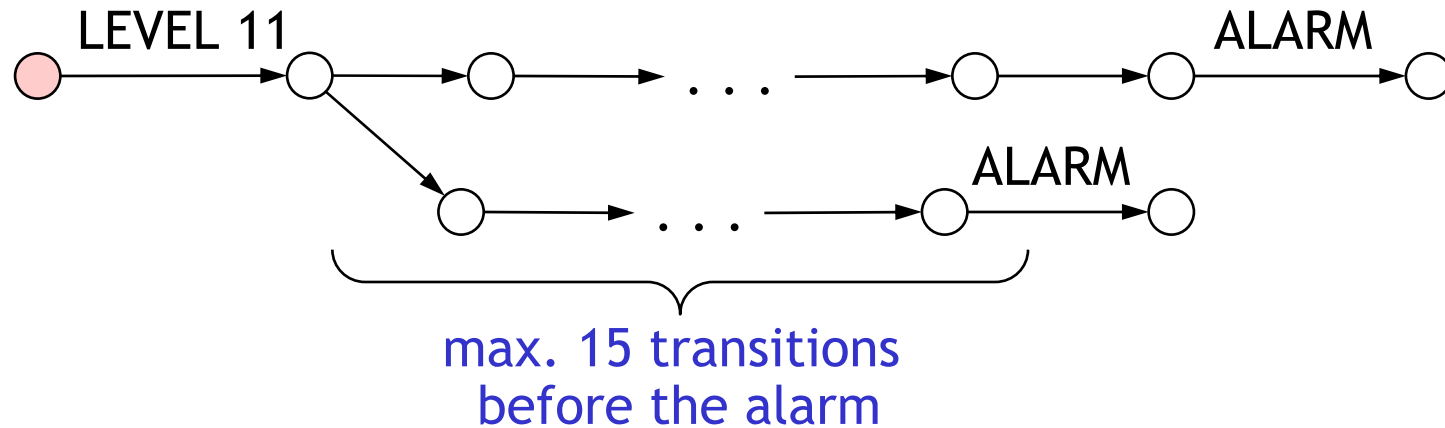
- (basic) syntax:



- $P$  contains « calls »  $X (E')$
- Allows to perform computations and store intermediate results while exploring the PLTS

# Example

- Counting of actions (e.g., clock ticks):



[ {LEVEL ?l:Nat where l > 10} ]

nu X (c:Nat := 15) .

[ not ALARM ] (c > 0 and X (c - 1))





# Quantifiers

- Existential quantifier:

exists  $x:T$  among  $\{ E_1 \dots E_2 \} . P$

limits of the subdomain of T

- Universal quantifier:

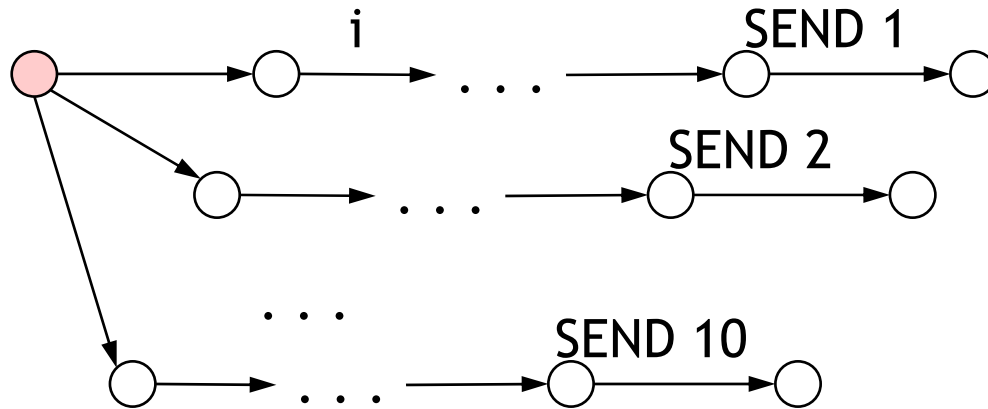
forall  $x:T$  among  $\{ E_1 \dots E_2 \} . P$

→ *shorthands for large disjunctions and conjunctions*



# Example

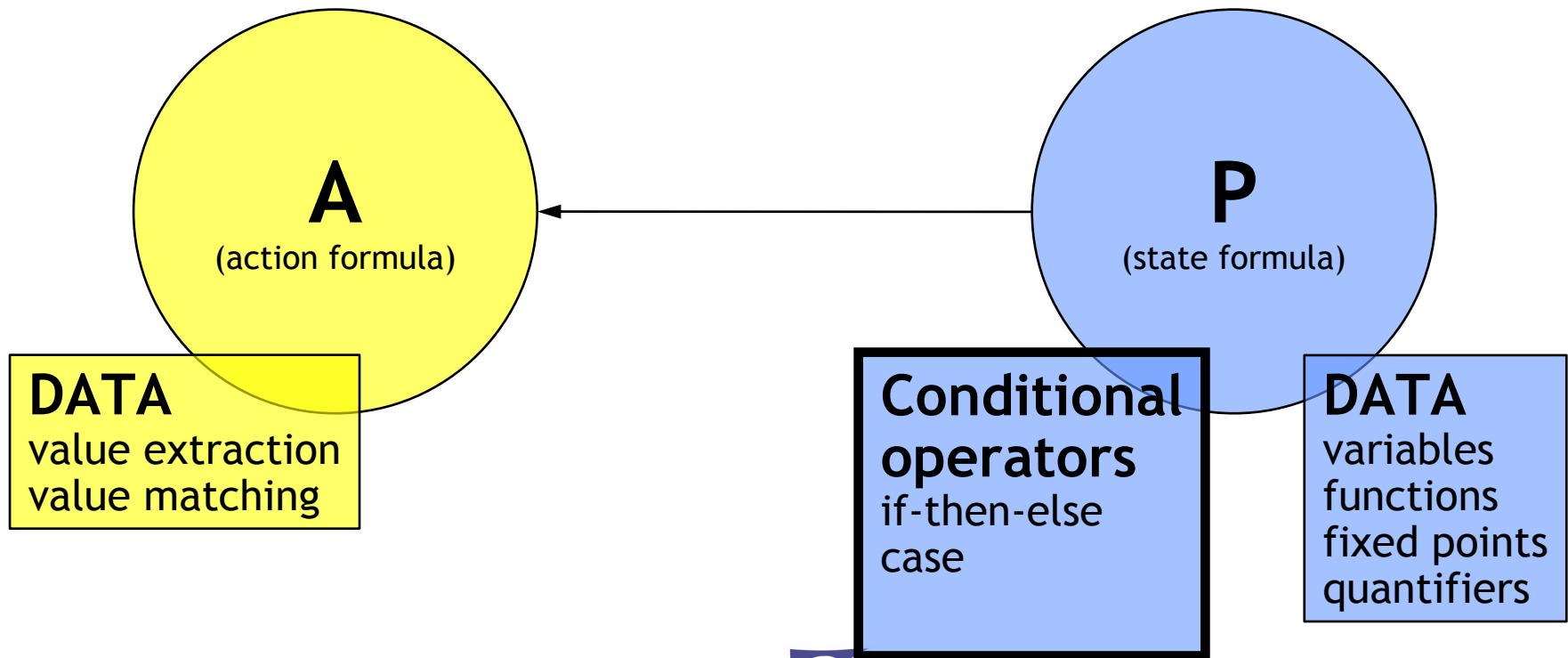
- Broadcast of messages:



forall msg:Nat among { 1 ... 10 } .

mu X . (< {SEND !msg} > true or < true > X)

# Model Checking Language (2/4)



# Conditional operators

- **Branching operator:**

```
if  $P_1$  then  $P_1'$ 
  elsif  $P_2$  then  $P_2'$ 
  ...
  else  $P_n'$ 
end if
```

propositionally  
closed subformulas in  
the branch conditions  
(to ensure syntactic  
monotonicity)

mandatory clause  
(to avoid exceptions)

- **Selection operator:**

```
case E is
   $M_1$  ->  $P_1$ 
  |
  | ...
  | any ->  $P_n$ 
end case
```

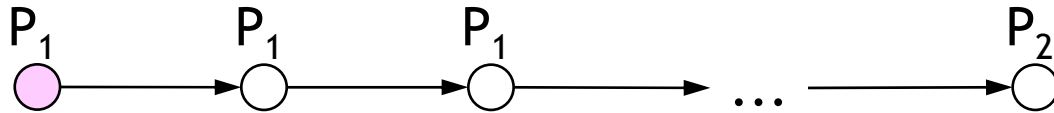
pattern

mandatory exhaustiveness  
(to avoid exceptions)



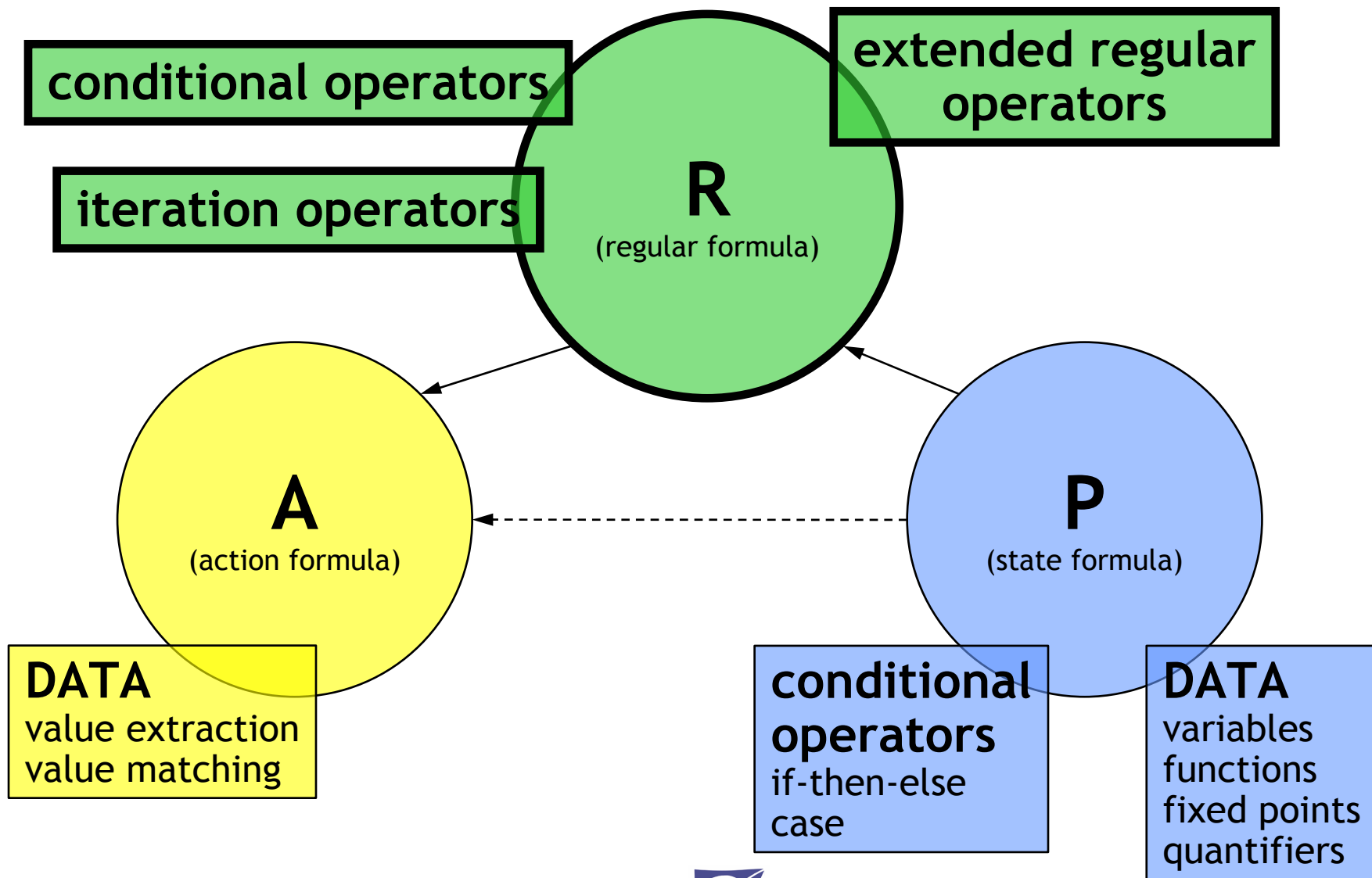
# Example

- Operator E ( $P_1 \text{ U } P_2$ ) of CTL:



$E (P_1 \text{ U } P_2) =$   
 $\text{mu } X . \text{ if } P_1 \text{ then } \langle \text{true} \rangle X \text{ else } P_2 \text{ end if}$

# Model Checking Language (3/4)



# Grammar extension

P ::= ...

| ~~[ A ] P~~

| ~~< A > P~~

| [ R ] P

| < R > P

| ...

regular formulas  
implemented by  
EVALUATOR 3.5

R ::= nil

| A

| R<sub>1</sub> . R<sub>2</sub>

| R<sub>1</sub> | R<sub>2</sub>

| R\*

| R+

| R?

| R { n }

| R { m ... n }

| R { m ... }

*empty sequence*

*one-step sequence*

*concatenation*

*choice*

*0 or more occurrences*

*1 or more occurrences*

*choice (0 or 1 occurrences)*

*exactly n occurrences*

*between m and n occurrences*

*at least m occurrences*

| if P<sub>1</sub> then R<sub>1</sub> ... else R<sub>2</sub> end if

| case E is M<sub>1</sub> -> R<sub>1</sub> ... end case

| while P do R end while

| repeat R until P end repeat



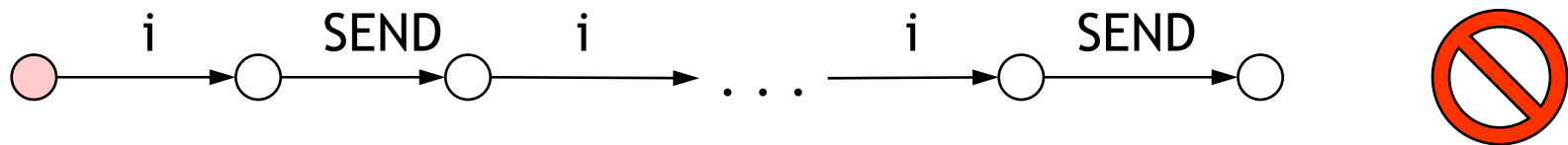
# Basic regular formulas (Evaluator 3.5)

- **Liveness** properties (existence of sequences):



$\langle \text{SEND} \cdot (i^* \cdot \text{ERROR} \cdot i^* \cdot \text{RETRY})^* \cdot i^* \cdot \text{RECV} \rangle \text{ true}$

- **Safety** properties (interdiction of sequences):



$[ \text{true}^* \cdot \text{SEND} \cdot (\text{not RECV})^* \cdot \text{SEND} ] \text{ false}$



# Conditional operators

- **Branching operator:**

if  $P_1$  then  $R_1$   
  elsif  $P_2$  then  $R_2$   
  ... else  $R_n$

propositionally  
closed subformulas in  
the branch conditions  
(to ensure syntactic  
monotonicity)

end if

- **Selection operator:**

case  $E$  is

$M_1 \rightarrow R_1 \mid \dots \mid M_n \rightarrow R_n$

end case

optional  
exhaustiveness  
(missing branches  
equivalent to nil)



# Counting operators

- $R \{ E \}$       *repetition E times*
- $R \{ E_1 \dots \}$       *repetition at least  $E_1$  times*
- $R \{ E_1 \dots E_2 \}$       *repetition between  $E_1$  and  $E_2$  times*

- Some identities:

$$\text{nil} = \text{false}^*$$

$$R + = R \cdot R^*$$

$$R^* = R \{ 0 \dots \}$$

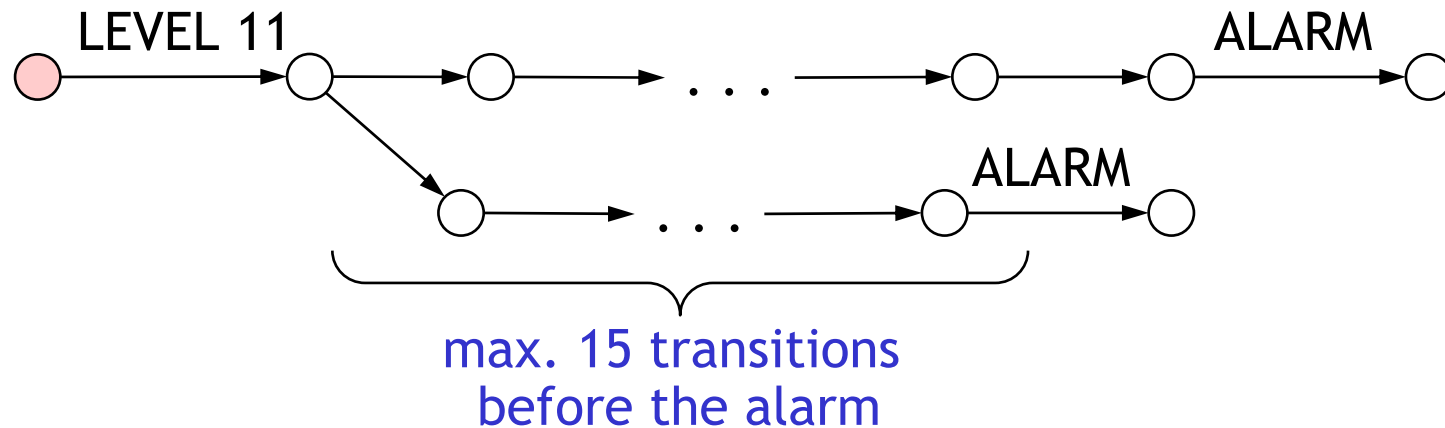
$$R? = R \{ 0 \dots 1 \}$$

$$R + = R \{ 1 \dots \}$$

$$R \{ E \} = R \{ E \dots E \}$$



# Example: action counting (revisited)



- Formulation using counting operators:

[ {LEVEL ?l:Nat where l > 10} .

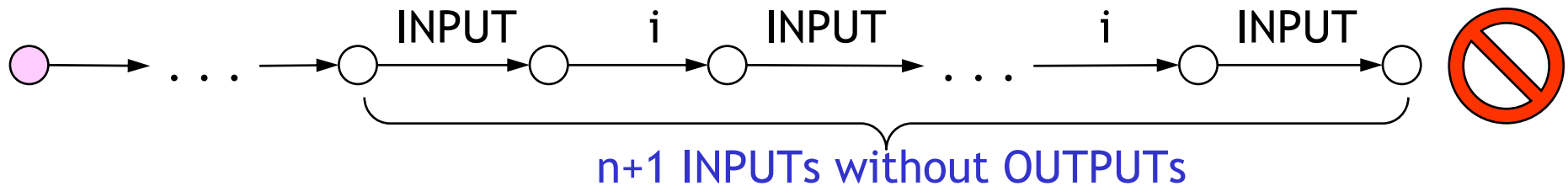
(not ALARM) { 16 } ] false



# Example: safety of a n-place buffer

- Formulation using extended regular operators:

$[ \text{true}^* \cdot ((\text{not OUTPUT})^* \cdot \text{INPUT}) \{ n + 1 \} ] \text{false}$



- Formulation using parameterized fixed points:

$\text{nu } X \cdot (\text{nu } Y \text{ (c:Nat:=0)} \cdot ($   
     $[\text{not OUTPUT}] Y \text{ (c)} \text{ and}$   
     $\text{if } c = n+1 \text{ then } [\text{INPUT}] \text{false}$   
     $\text{else } [\text{INPUT}] Y \text{ (c+1)}$   
     $\text{end if})$   
 $\text{and } [\text{true}] X)$



# Iteration operators

- Cycle with initial test:

while P do

R

end while

- Cycle with final test:

repeat

R

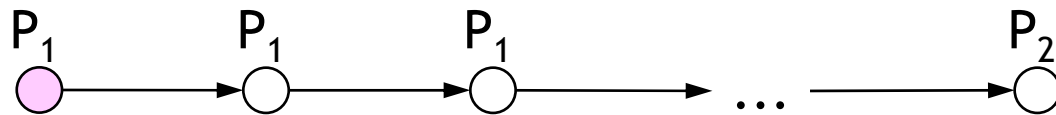
until P end repeat

propositionally  
closed subformulas in  
the cycle conditions  
(to ensure syntactic  
monotonicity)



# Example

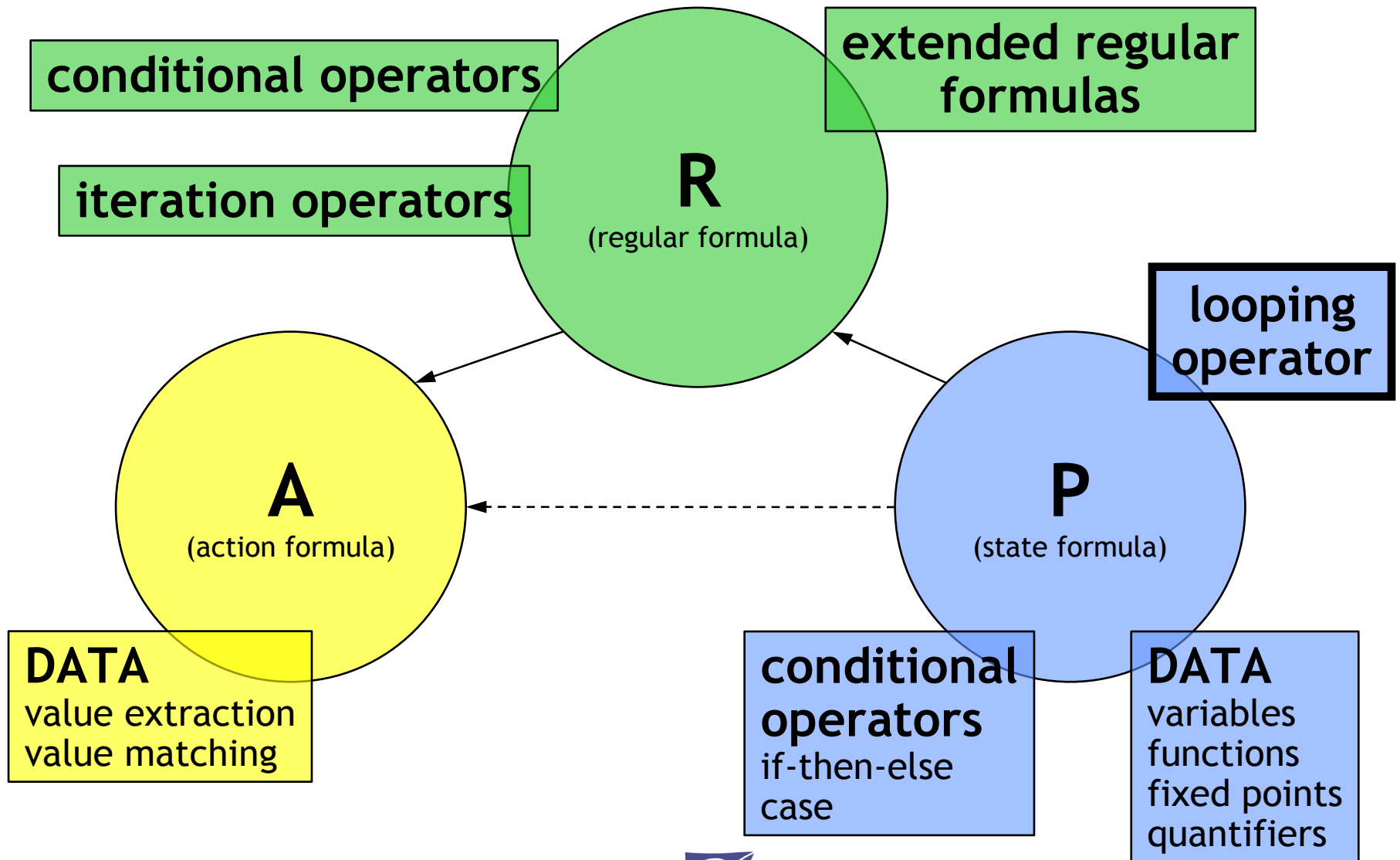
- Operator E ( $P_1 \text{ U } P_2$ ) of CTL (revisited):



$E (P_1 \text{ U } P_2) =$   
< while  $P_1$  and not  $P_2$  do  
    true  
end while >  $P_2$



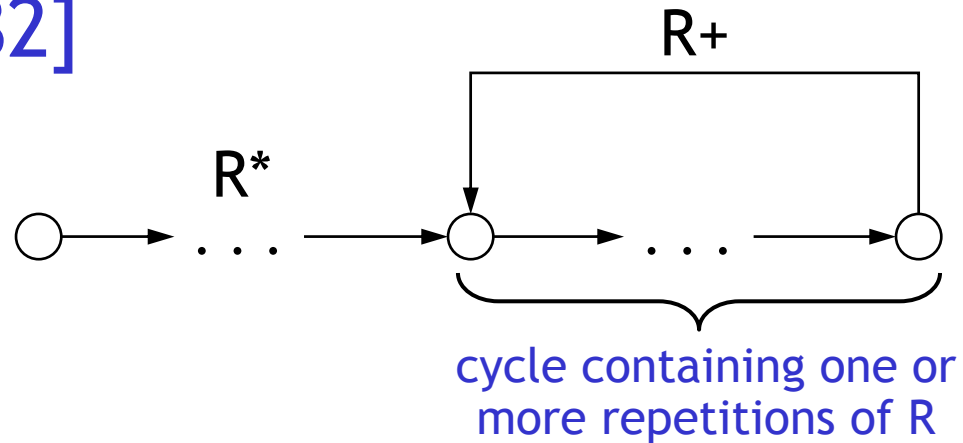
# Model Checking Language (4/4)



# Looping operator (from PDL-delta)

- $\Delta R$  operator added to PDL to specify infinite behaviours [Streett-82]

- MCL syntax:  $\langle R \rangle @$



- Examples:

- process overtaking

$[REQ_0] \langle (not\ GET_0)^* \cdot REQ_1 \cdot (not\ GET_0)^* \cdot GET_1 \rangle @$

- Büchi acceptance condition

$\langle true^* \cdot if\ P_{accepting}\ then\ true\ end\ if \rangle @$

$\rightarrow$  allows to encode LTL model checking





# On-the-fly verification

## • Principle

- Translate the MCL formula into a (parameterised) HMLR equation system
- Translate the verification of a HMLR system on a PLTS into a PBES resolution
- Expand the PBES into a plain BES on-the-fly
- Solve the BES locally ([Caesar\\_Solve](#) library of CADP)

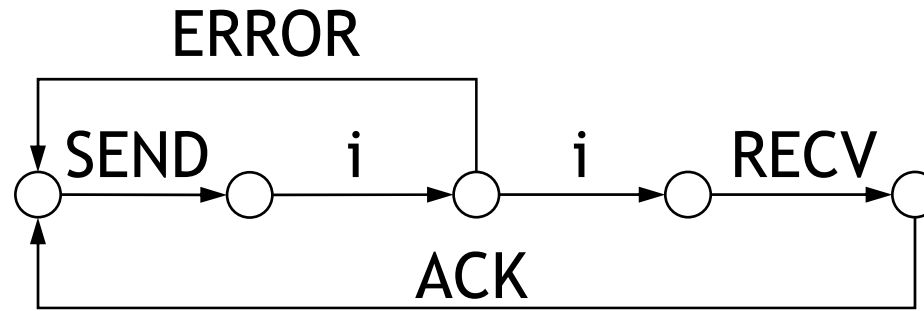
## • Optimizations

- Operators of CTL, ACTL, PDL-delta
  - Compiled into disjunctive/conjunctive BESs
  - Memory-efficient algorithms A3 and A4 [[Mateescu-03](#)]

## • Diagnostic generation



# Example



- Every SEND is followed by a RECV after 2 steps:

$[ \text{true}^* . \text{SEND} ] < \text{true} \{ 2 \} . \text{RECV} > \text{true} =$   
 $\text{nu } X . ( [ \text{SEND} ] \text{mu } Y (c:\text{Nat} := 2) .$

$\text{if } c = 0 \text{ then } < \text{RECV} > \text{true}$   
 $\text{else } < \text{true} > Y (c - 1)$   
 $\text{end if}$

and

$[ \text{true} ] X )$

# Translation into HMLR

$\text{nu } X . [ \text{SEND} ]$

and  $[ \text{true} ] X$

$\text{mu } Y (c:\text{Nat} := 2) .$

if  $c = 0$  then  $\langle \text{RECV} \rangle \text{true}$

else  $\langle \text{true} \rangle Y (c - 1)$

end if

$\{ X =_{\text{nu}}$

$[ \text{SEND} ] Y (2)$

and

$[ \text{true} ] X$

$\}$

$\{ Y (c:\text{Nat}) =_{\text{mu}}$

if  $c = 0$  then  $\langle \text{RECV} \rangle \text{true}$

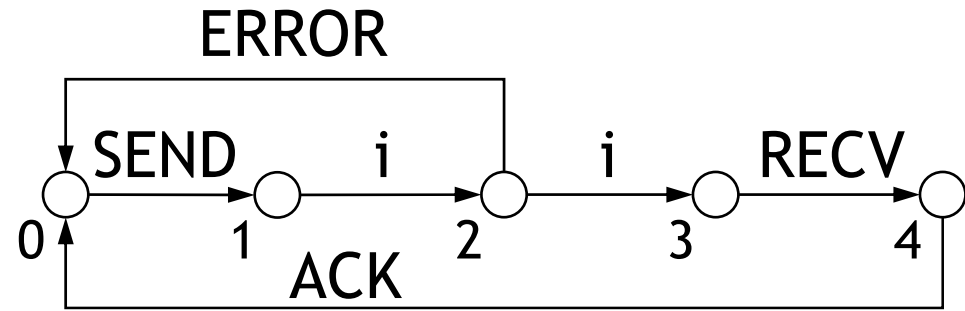
else  $\langle \text{true} \rangle Y (c - 1)$

end if

$\}$



# Translation into BESs + resolution



```

{ X =nu
  [ SEND ] Y (2)
  and
  [ true ] X
}

```

```

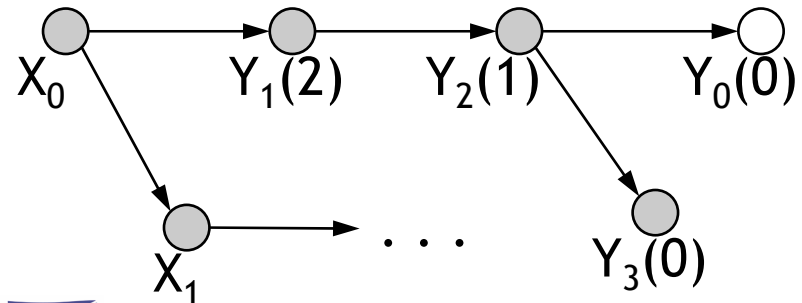
{ Y (c:Nat) =mu
  if c = 0 then < RECV > true
  else < true > Y (c - 1)
  end if
}

```

- Encoding scheme:

$$X_s = \text{“}s \models X\text{”}$$

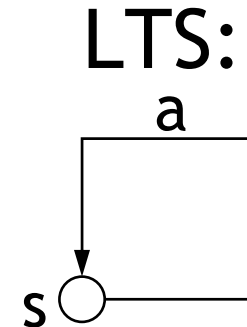
$$Y_s (c) = \text{“}s \models Y (c)\text{”}$$



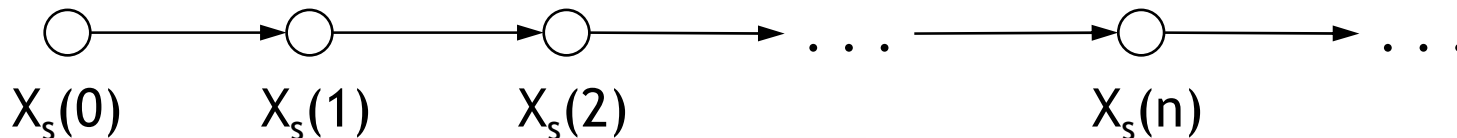
# Divergence

- In presence of data parameters of infinite types, termination of model checking is not guaranteed anymore
- (pathological) property:

$\mu X (n:\text{Nat} := 0) . \langle a \rangle X (n + 1)$



BES :  $\{ X_s (n:\text{Nat}) =_{\mu} \text{OR } s \xrightarrow{a} s', X_{s'} (n + 1) \} =$   
 $\{ X_s (n:\text{Nat}) =_{\mu} X_s (n + 1) \}$



# Linear-time model checking (looping operator)

- Translation in mu-calculus of alternation depth 2 [Emerson-Lei-86]:

$$\langle R \rangle @ = \nu X . \langle R \rangle X$$

if R contains \*-operators,  
the formula is of  
alternation depth 2

- But still checkable in linear-time:
  - Mark LTS states potentially satisfying X
  - Computation of SCCs containing marked states
    - Can serve for LTL model checking
    - Allows linear-time handling of repeated invocations
  - $A4_{cyc}$  algorithm for local BES resolution



# Model checking complexity

- Formulas **without** data:

- Linear-time w.r.t. the PLTS and formula size
- Problem rephrased in terms of a BES
- On-the-fly resolution
  - Linear algorithms of the [Caesar\\_Solve](#) library [[Mateescu-06](#)]
  - New linear algorithm  $A4_{cyc}$  for the looping operator

- Formulas **with** data:

- Additional complexity depending on the number of actual parameter values computed during verification
- Risk of divergence (same as for recursive functions in programming languages; however cycles are allowed)
- All counting regular operators are convergent



# EVALUATOR 4.0

## • Implemented in CADP

- Benefits from OPEN/CAESAR environment: interface with languages (LOTOS, FSP...) and formats (AUT, EXP...) supported by CADP

## • Front-end

- Evaluation of an MCL formula on a PLTS is translated by 6 successive phases into a parameterized boolean equation system (PBES)

## • Back-end

- PBES is expanded into a BES and solved on-the-fly (with the Caesar\_Solve library of CADP)

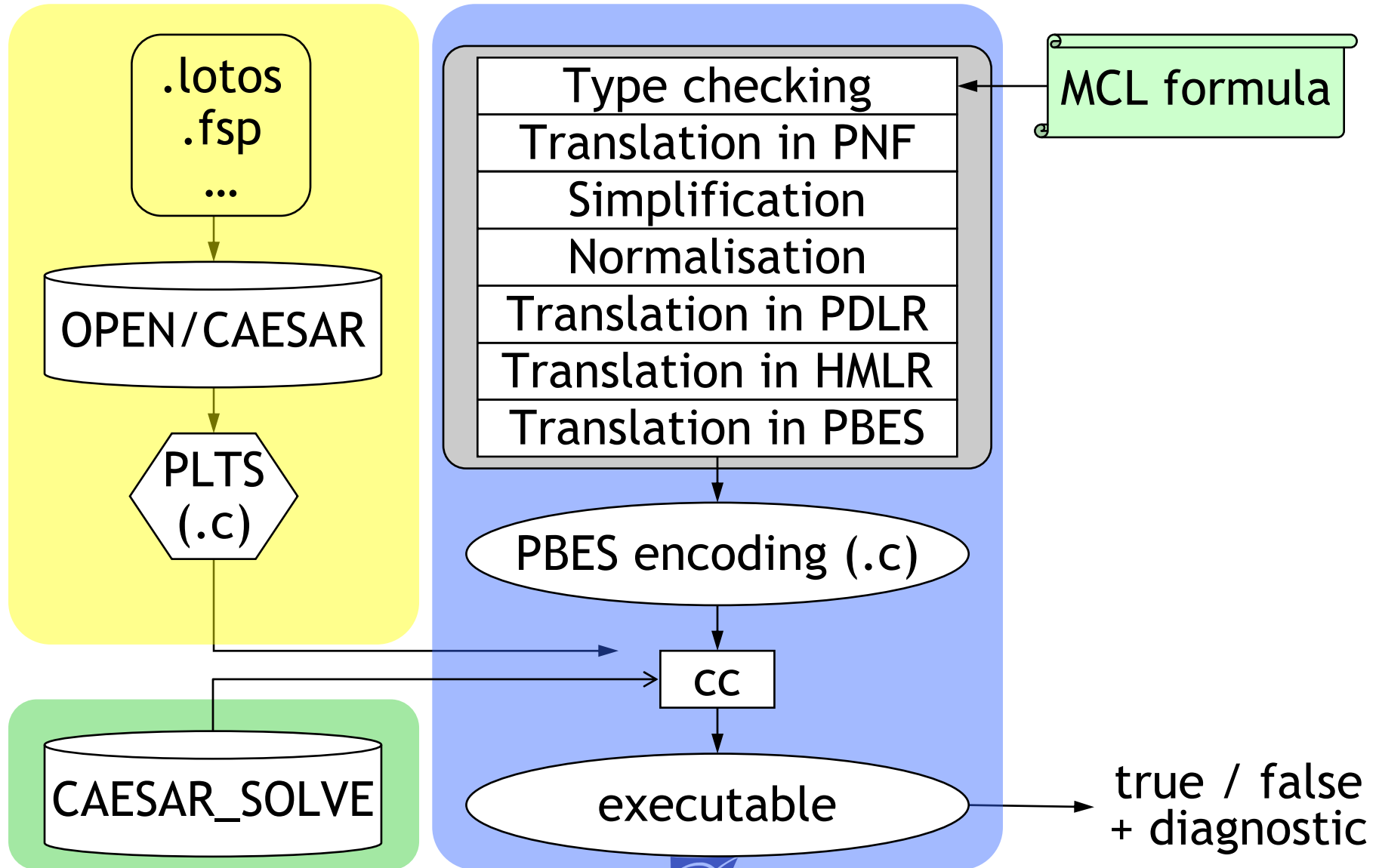
## • Stats

- 50,000 lines of code (Syntax, LOTOS NT, C)

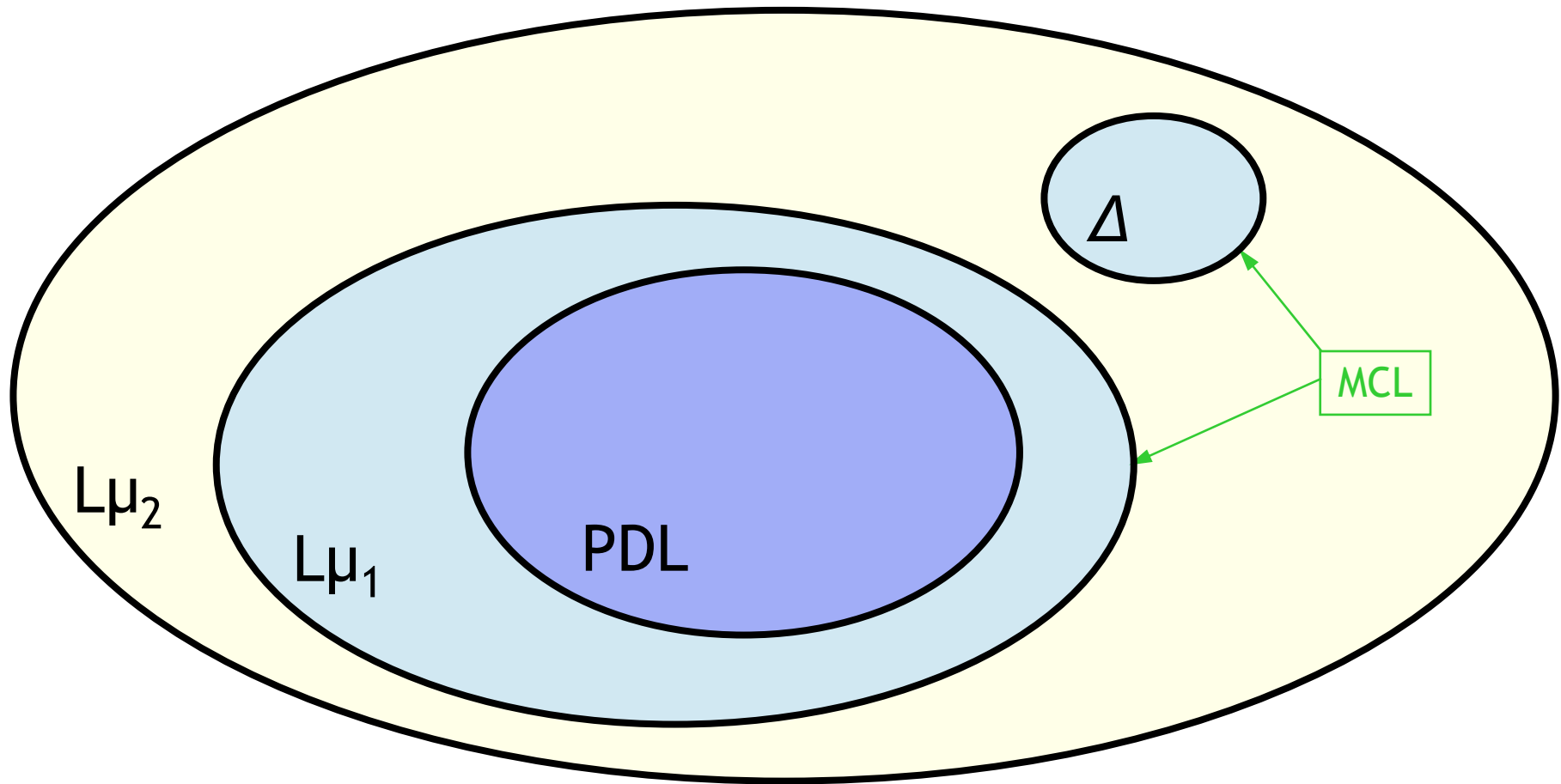




# EVALUATOR 4.0



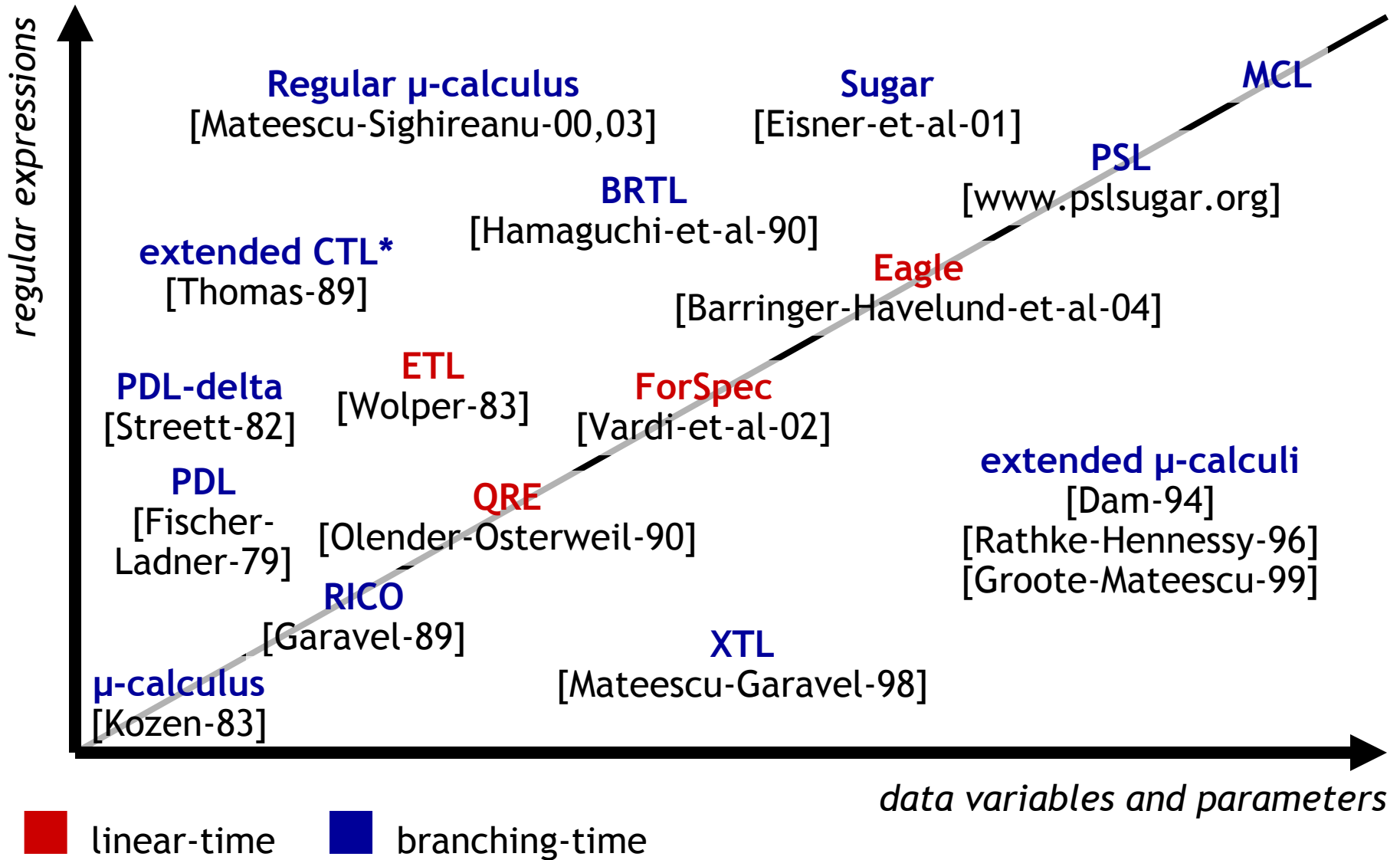
# MCL expressiveness (dataless part)



$CTL^* \subseteq PDL \cup \Delta \subseteq MCL$   
[Wolper-82]



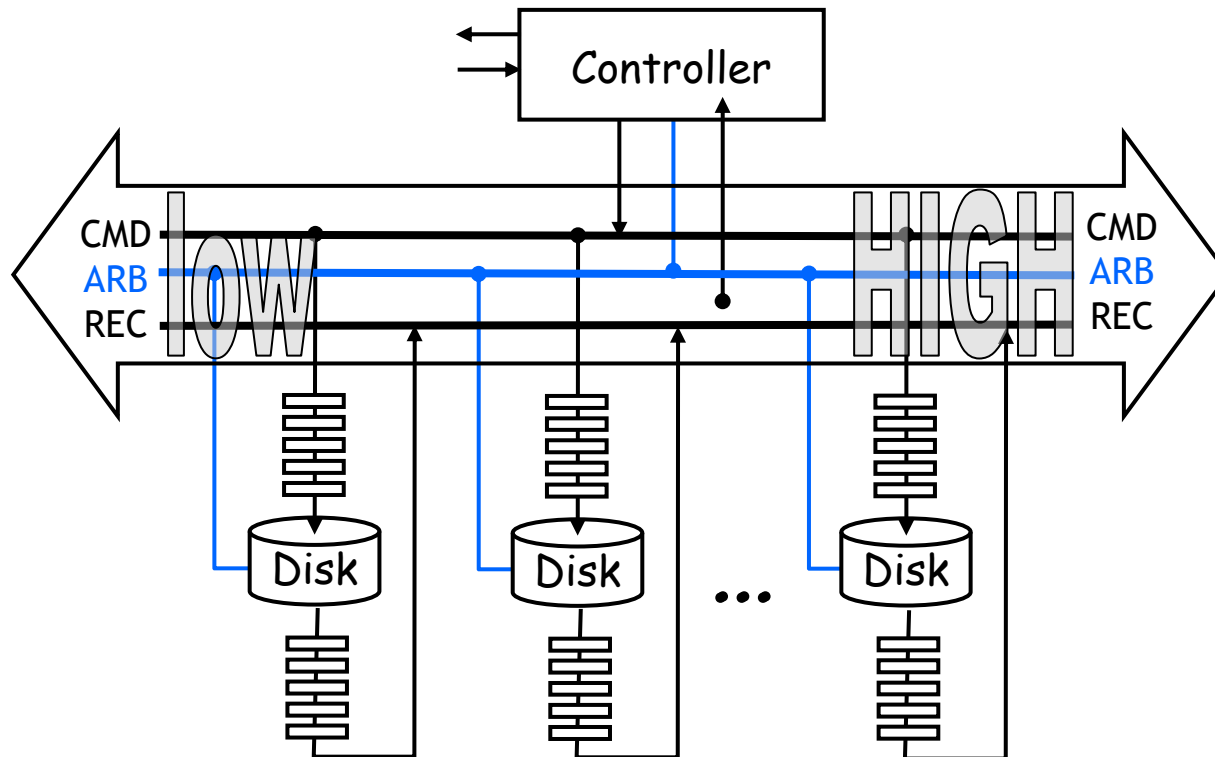
# Related work (TL extensions)



# Demo

## (SCSI-2 bus arbitration protocol)

- **Prioritized** arbitration mechanism, based on static IDs on bus (devices numbered from 0 to  $n - 1$ )
- **Fairness** problem (starvation of low-priority disks)



# Starvation property

“Every time a disk  $i$  with priority lower than the controller  $nc$  receives a command, its access to the bus can be continuously preempted by any other disk  $j$  with higher priority”

[ true\*. {cmd ?i:Nat where i < nc} ]

forall j:Nat among { i + 1 ... n - 1 } .

(j <> nc) implies

< (not {rec !i})\*. {cmd !j} .

(not {rec !i})\*. {rec !j} > @



# Safety property

*“The difference between the number of commands received and reconnections sent by a disk  $i$  varies between 0 and 8 (the size of the buffers associated to disks)”*

```
forall i:Nat among { 0 ... n - 1 } .  
  nu Y (c:Nat:=0) . (  
    [ {cmd !i} ] ((c < 8) and Y (c + 1))  
    and  
    [ {rec !i} ] ((c > 0) and Y (c - 1))  
    and  
    [ not ({cmd !i} or {rec !i}) ] Y (c)  
  )
```



# Conclusion and future work

- **Already available:**

- Formal definition of the MCL language
  - Syntax and semantics
  - Translation into more primitive forms
- EVALUATOR 4.0
  - Currently under testing, integrated in the next CADP release
  - Used in case studies with BULL (multiprocessor architectures) and EC-MOAN European project (bioinformatics)
  - Linear time model checking (dataless part), stores only states

- **Ongoing and future:**

- Packaging EVALUATOR 4.0 (testing and demos)
- Allow types and functions definition
- Interface with LOTOS-defined data types

