Efficient Algorithms for Three Reachability Problems in Safe Petri Nets

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Three reachability problems

- We focus on ordinary safe Petri Nets
- Dead Places Problem:
  - a place is dead if it never gets a token
  - for each place p, decide $\neg R(\{p\})$, where
    $R(M) \stackrel{\text{def}}{=}$ *there exists a reachable marking in which M is included*
- Dead Transitions Problem:
  - a transition is dead if it is never enabled
  - for each transition t, decide $\neg R({}^{\bullet}t)$
- Concurrent Places Problem:
  - two places are concurrent if they can have a token simultaneously
  - for each two places p_1 and p_2, decide $R(\{p_1, p_2\})$
  - concurrency between places is symmetric and quasi-reflexive

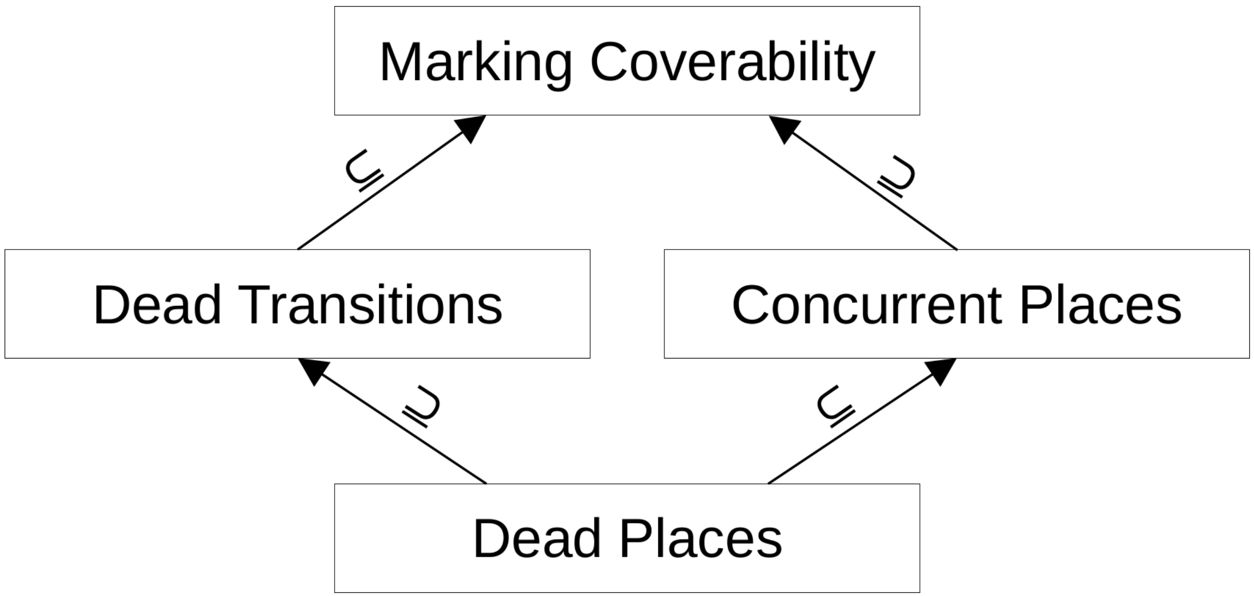
Why are these problems interesting?

- Dead places and dead transitions:
  - useful for simplifying complex Petri nets, especially those generated from higher-level formalisms
  - profitable reduction: 20.4% dead places and 37.7% dead transitions in practice

- Concurrent places:
  - crucial role for the decomposition of Petri nets into automata networks [Bouvier et al., Petri Nets 2020]
  - statistically: 67% non-concurrent pairs of places

Complexity of these problems

- 3 subproblems of the Marking Coverability Problem:



- These 3 problems are PSPACE-complete

What about non-safe Petri nets?

- Concurrent places are most interesting on safe Petri nets

- For any state machine having no dead place:
  - 1 initial token

    $\Rightarrow$ each place is only concurrent with itself
  - 2 initial tokens

    $\Rightarrow$ all places are pairwise concurrent

Practical motivation

- Despite PSPACE complexity, we seek for efficient algorithms that solve a majority of problems
- Benchmarks:
  - we use a collection of 13,116 nets from academia, industry, and competitions
  - these models are diverse and complex

property	yes	no
pure	62.9%	37.1%
free choice	41.3%	58.7%
extended free choice	42.7%	57.3%
marked graph	3.5%	96.5%
state machine	12.1%	87.9%

property	yes	no
connected	94.0%	6.0%
strongly connected	14.3%	85.7%
conservative	16.5%	83.5%
sub-conservative	29.7%	70.3%
non trivial and unit safe	67.7%	32.3%

feature	min value	max value	average	median	std deviation
#places	1	131,216	282.4	15	2690
#transitions	0	16,967,720	9232.8	20	270,287
#arcs	0	146,528,584	72,848	55	2,141,591
arc density	0.0%	100.0%	14.5%	9.4%	0.2

Straightforward approach

- Reuse existing model checkers for Petri nets:
  - encode the 3 problems as temporal-logic formulas
  - analyse model-checking results to get dead places, dead transitions, and concurrent places

- Possible, yet inefficient:
  - linear or quadratic number of formulas
    (300 places $\Rightarrow$ 45,150 formulas for concurrent places)
  - redundant calculations: many similar formulas need to be evaluated on the same Petri net

Dedicated approach

- Instead, we suggest tools with built-in options:
  - option -dead-places
    result = vector of {dead, non-dead, unknown} values
    indexed by place numbers
  - option -dead-transitions
    result = vector of {dead, non-dead, unknown} values
    indexed by transition numbers
  - option -concurrent-places
    result = half-matrix of {concurrent, non-concurrent, unknown}
    values, indexed by place numbers

```
1
01      <- (p2, p1) non-concurrent
011
0101    <- (p4, p4) concurrent
01011
000111
.101001 <- (p7, p1) unknown
```

Algorithms for computing the vectors of dead places and dead transitions

1. Marking graph exploration

- Explore all reachable markings, e.g., using decision diagrams
  - PSPACE-complete $\Rightarrow$ may take too long or too much memory
- Algorithmic enhancements:
  - timeout or limit on exploration depth
  - speed-up calculations by not firing already known dead transitions
  - shortcuts: halt exploration as soon as all results are known
- Expected results, for all places and transitions:
  - complete exploration: gives dead or non-dead values
  - incomplete exploration: gives non-dead or unknown values
    in this case, we apply additional algorithms to remove as many unknown values as possible

2. Structural rules

- 8 simple theorems to compute some dead or non-dead values:

  – Any place belonging to the initial marking M_0 is not dead.
  – Any transition having no input place (and no output place) is not dead.
  – If a place p is dead, all the transitions of ${}^\bullet p \cup p^\bullet$ are also dead.
  – If a transition t is not dead, all the places of ${}^\bullet t \cup t^\bullet$ are also not dead.
  – If a transition t is dead, any place p such that ${}^\bullet t = \{p\}$ is also dead.
  – If a place p is not dead, any transition t such that ${}^\bullet t = \{p\}$ is also not dead.
  – If the net is safe, any transition whose input places form a strict subset of the output places is dead.
  – If the net is unit safe, any transition having at least two input (resp. two output) places located in two non-disjoint NUPN units is dead.

- These rules are applied repeatedly until saturation

3. Linear over-approximation

- Abstraction:
  - the set of reachable markings is replaced by a set E of places such that, at the end of the algorithm:
    $p \notin E \;\Rightarrow$ place p is dead
    ${}^\bullet t \not\subseteq E \;\Rightarrow$ transition t is dead
- Algorithm:
  - initially, E is the initial marking
  - repeat until saturation: for each t, ${}^\bullet t \subseteq E \Rightarrow t^\bullet \subseteq E$
- This gives, for each place and transition, either a dead or an unknown value

Combination of approaches

- Approaches 1-3 are combined in a well-chosen order:
  - structural rules
  - linear over-approximation
  - marking graph exploration
  - structural rules (again)

- Two implementations:
  - Caesar.bdd: 11K lines of C code (using Cudd for BDDs)
  - ConcNUPN: 730 lines of Python

  ConcNUPN is used to cross-check results of Caesar.bdd

Experimental results using Caesar.bdd

problem	value of t	0	5	10	15	30	45	60	120	180	240	300
dead places	% complete vectors	44.6	93.0	93.6	93.8	94.4	94.6	95.1	95.3	95.4	95.5	95.6
	% unknowns values	48.9	33.5	32.0	31.3	28.9	28.3	27.9	27.1	26.5	25.9	25.8
	% vector completion	69.3	97.0	97.3	97.5	97.7	97.9	97.9	98.1	98.1	98.2	98.2
dead trans.	% complete vectors	29.3	92.3	92.9	93.2	93.7	94.0	94.1	94.4	94.7	94.9	95.0
	% unknowns values	68.7	65.0	63.5	62.0	61.0	59.3	57.8	54.6	45.2	39.9	29.8
	% vector completion	50.9	95.8	96.2	96.4	96.7	96.8	96.9	97.1	97.2	97.3	97.3

- Dead places (with a BDD timeout of 60 s):
  - fully solved vectors (no unknown values): 95.1%
  - average number of unknown values in vectors: 2.1%
- Dead transitions (with a BDD timeout of 60 s):
  - fully solved vectors (no unknown values): 94.1%
  - average number of unknown values in vectors: 3.1%

Algorithms for computing the half matrix of concurrent places

1. Marking graph exploration

- First, explore reachable markings, e.g., using decision diagrams:
  - PSPACE-complete: the exploration may be incomplete (timeout or limit on exploration depth)
  - shortcuts are impossible or very unlikely, contrary to the marking graph exploration for dead places
- Then, check for all pairs of places whether it exists a reachable marking containing these places
- Expected results for all pairs of places:
  - complete exploration:
    gives concurrent or non-concurrent values
  - incomplete exploration:
    gives concurrent or unknown values

2. Structural rules

- 8 theorems giving concurrent or non-concurrent pairs:
  - The places of the initial marking M_0 are pairwise concurrent.
  - If a transition is not dead, its input places (resp. output places) are pairwise concurrent.
  - A non dead place is concurrent with itself.
  - A dead place is non concurrent with any other place, including itself.
  - If a dead transition has two (distinct) input places, these places are non concurrent.
  - If a transition t (dead or not) has a single input place p, this place is non concurrent with any output place of t different from p.
  - For any path $(p_1, t_1, p_2, t_2, ..., p_n, t_n, p_{n+1})$ such that each transition t_i has a single input place p_i and at least one output place p_{i+1}, the places p_1 and p_{n+1} are non concurrent if they are distinct.
  - If the net is a unit-safe NUPN, any two distinct places located in non-disjoint units are non concurrent. In particular, any two distinct places located in the same unit are non concurrent.
- These rules are applied until saturation, together with theorems for dead places and dead transitions

3. Quadratic under-approximation

- Abstraction: the set of reachable markings replaced by a set E of pairs of places such that $\{p_1, p_2\} \in E \Rightarrow p_1$ and p_2 concurrent
- 4 theorems repeated until saturation:
  - If a place p is not dead, any transition t such that $^\bullet t = \{p\}$ is also not dead.
  - If two (distinct) places p_1 and p_2 are concurrent, any transition t such that $^\bullet t = \{p_1, p_2\}$ is not dead.
  - If a transition is not dead, its output places are pairwise concurrent.
  - If two distinct places p_1 and p_2 are concurrent, p_2 is also concurrent with each output place of any transition t such that $^\bullet t = \{p_1\}$.
- This gives certain pairs of concurrent places

4. Quadratic over-approximation

- Generalizes a former algorithm [Kovalyov & Esparza, 1996]
- Abstraction: the set of reachable markings is replaced by a set E of pairs of places, such that, at the end of the algorithm: p_1 and p_2 concurrent $\Rightarrow \{p_1, p_2\} \in E$
- Algorithm:
  - operator $M_1 \otimes M_2 \stackrel{\text{def}}{=} \{\{p_1, p_2\} \mid p_1 \in M_1 \wedge p_2 \in M_2\}$
  - auto-product $M^{②} \stackrel{\text{def}}{=} M \otimes M$
  - initially $E = M_0^{②}$, where M_0 is the initial marking
  - repeat until saturation: for each transition t,
    for each set of places M: ${}^\bullet t \subseteq M \wedge M^{②} \subseteq E \Rightarrow ((M \setminus {}^\bullet t) \cup t^\bullet)^{②} \subseteq E$
- This gives, for each pair of places, either a non-concurrent or an unknown value

Combination of approaches

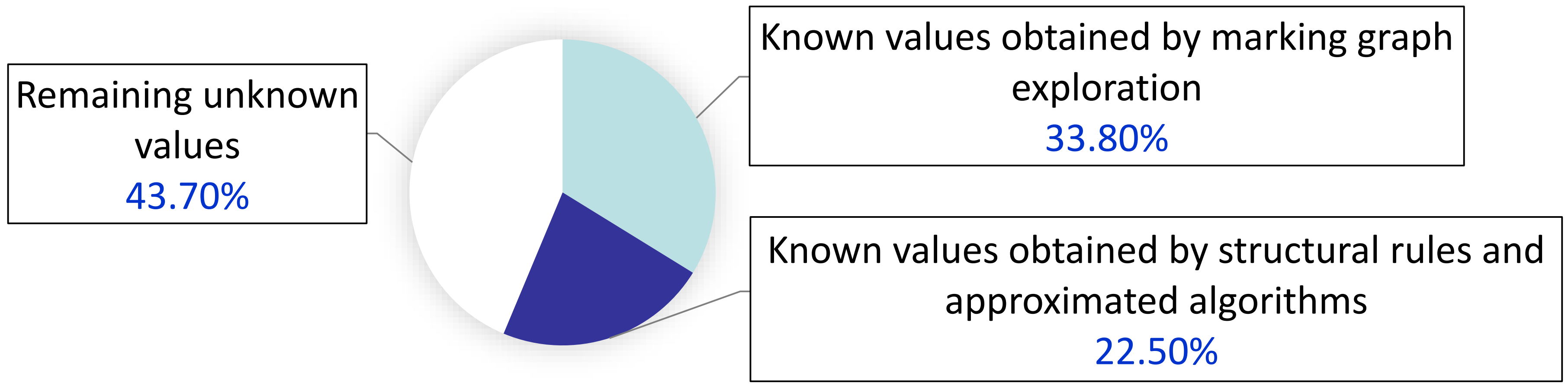
- Approaches 1-4 are combined in the following order:
  - marking graph exploration
  - structural rules
  - quadratic under-approximation
  - quadratic over-approximation

- Implemented in Caesar.bdd and ConcNUPN

Experimental results using Caesar.bdd

value of t	0	5	10	15	30	45	60	120	180	240	300	360	420
% complete matrices	51.0	91.6	92.2	92.5	93.0	93.6	94.0	94.2	94.4	94.5	94.6	94.7	94.7
% unknowns values	45.0	44.7	44.7	44.4	44.4	43.7	43.7	43.7	43.6	43.6	43.6	43.6	43.6
% matrix completion	81.6	96.3	96.6	96.8	97.0	97.1	97.2	97.3	97.4	97.4	97.4	97.5	97.5

- For a BDD timeout of 60 seconds:
  - fully solved matrices (no unknown values): 94%
  - average number of unknown values in matrices: 2.8%
- For the few incomplete half matrices:



Conclusion

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- Three useful, yet difficult problems (PSPACE-complete)
- Combination of approaches to handle large models:
  - ≈ 95% of models are completely solved (on 13,000+ nets)
  - some large models are partially solved (the solution contains unknown values)
- Future work: remove more unknown values using, e.g., invariants, partial orders, SAT solving, structural reductions, etc.
- Other tools are starting to address these problems:
  - Kong (Nicolas Amat, LAAS-CNRS) – structural reductions
  - ITS-Tools (Yann Thierry-Mieg, LIP6) – model checking